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MARAGONI CONVECTION IN A TWO-LAYER SYSTEM

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Equations are obtained that govern the characteristics of weakly nonlinear concentration and thermocapillary convection stabilized by the nonlinearity of convective transport of impurity mass and heat.

Under unstable surface tension conditions depend on the impurity concentration or temperature on the interface of two immiscible fluids, the appearance of the instability of the mass or heat transport process through this boundary (Marangoni effect) is possible. For plane interfacial surfaces such an instability was apparently first considered in [1-3], and was then investigated in a very large number of papers with diverse physicochemical factors and the curvature of the surface itself taken into account.

Ordered or chaotic convective motions (interphasal convection or turbulence) which are capable of significant intensification of the mass as well as heat transport through this surface [4-6], are formed as a result of the mentioned instability in domains adjacent to the interfacial surface. Thus, for instance, the effect mass transfer coefficient during extraction in liquid-liquid systems can be increased because of natural or artificially produced Marangoni convection by 2-10 times as compared with analogous stable systems [7].

If known numerical investigations of interphasal convective structures of the type performed in [8] are excluded from consideration, then attempts at a nonlinear analysis of the result of the appearance of the Marangoni instability, which are perfectly necessary for estimation of the parameters of these structures, are limited, in practice, to weakly nonlinear problems to which the method of the small parameter developed in [9-11] is applicable. However even in this case the calculations and final equations turn out to be quite awkward, which requires their asymptotic and numerical analysis for comparatively simple particular situation.

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In this effort, in particular, it turns out to be expedient to consider separately [11] the possible nonlinearities in the system that result in stabilization of the perturbations that grow because of the instability.

We here assume that only the specific nonlinearity for the convective diffusion of heat conduction equations is essential while all the remaining nonlinearities can be neglected in a first approximation. This means that the interfacial surface should be considered imperturbable within the framework of the appropriate problem, the surface tension coefficient should be considered as a linear function of the surface concentration or temperature, and the liquid motion in the domains on both sides of the interfacial surface should be described in the Stokes approximation. Such a situation is perfectly realistic if the Schmidt (Prandtl) number is large compared with one, the dependence of σ on γ or T_S can be approximated sufficiently accurately by a linear function and the pressure perturbations at the surface, due to the origination of convective structures, are less in order of magnitude than the characteristic surface pressure σk .

For definiteness, we consider first the concentration-capillary convection near the surface $z = 0$ that separates a finite thickness liquid layer into upper and lower half-spaces. We consider the thickness of both layers large as compared with the characteristic dimension (period) of the convective structure, which permits neglecting exponentially small terms in the calculations and we consider just structures formed by cylindrical "shafts" which permits us to limit the analysis to plane hydrodynamics and convective diffusion problems. (Such bandlike cells, similar to unimodal convective, cells are ordinarily observed for a shallow setting in the instability domain [12]). Later the results obtained for the concentration-capillary convection are also extended to thermocapillary convection.

Below we consider the state of rest in the presence of mass flow normal to the interfacial surface

$$Q^\circ = D_1 \beta_1 = D_2 \beta_2, \quad (1)$$

as unperturbed, so that the impurity concentration in the liquid layers is

$$c_j^\circ = c_{sj}^\circ - \beta_j z, \quad j = 1, 2. \quad (2)$$

The unperturbed surface concentration equals

$$\gamma^\circ = m_1 c_{s1}^\circ = m_2 c_{s2}^\circ, \quad (3)$$

i.e., the volume concentrations on both sides of the interfacial surface do not certainly agree. The coefficients m_j in (3) are determined by the thermodynamic equilibrium conditions of the dissolved substance in the bulk phases and in a thin (practically monomolecular) layer on the interfacial surface. We approximate the surface tension coefficient by the function

$$\sigma = \sigma(\gamma) = \sigma^\circ - a\gamma, \quad (4)$$

where $a > 0$ for surfactants, and $a < 0$ for inactive substances. In conformity with the Atvesh formula, the coefficient a is positive for an analogous dependence of σ on the surface temperature.

The linear Stokes equations and its boundary conditions, with (4) taken into account, result in the following problem for the stream function [11]

$$\begin{aligned} (v_j \Delta - \partial/\partial t) \Delta \psi_j &= 0; \quad \psi_1 \rightarrow 0, \quad z \rightarrow \infty; \quad \psi_2 \rightarrow 0, \quad z \rightarrow -\infty; \\ \frac{\partial \psi_1}{\partial x} &= \frac{\partial \psi_2}{\partial x} = 0; \quad \frac{\partial \psi_1}{\partial z} = \frac{\partial \psi_2}{\partial z}; \quad z = 0; \\ \mu_2 \left(\frac{\partial^2 \psi_2}{\partial z^2} - \frac{\partial^2 \psi_2}{\partial x^2} \right) &- \mu_1 \left(\frac{\partial^2 \psi_1}{\partial z^2} - \frac{\partial^2 \psi_1}{\partial x^2} \right) = -a \frac{\partial \gamma}{\partial x}, \quad z = 0 \end{aligned} \quad (5)$$

(here h_1 and h_2 tended to infinitely; as is easy to show, the errors originating here are exponentially small).

The convective diffusion equation in the liquid layers and the corresponding boundary conditions are

$$\begin{aligned} \partial c_j / \partial t + v_j \nabla (c_j^0 + c_j) &= D_j \Delta c_j; \quad c_1 = 0, \quad z = h_1; \quad c_2 = 0, \quad z = -h_2; \\ m_1 c_1 &= m_2 c_2 = \gamma; \quad z = 0; \end{aligned} \quad (6)$$

$$\frac{\partial \gamma}{\partial t} + \frac{\partial}{\partial x} [(\gamma^0 + \gamma) v_x] = D_1 \frac{\partial c_1}{\partial x} - D_2 \frac{\partial c_2}{\partial x}; \quad z = 0.$$

The last boundary condition is substantially the impurity mass conservation equation in the surface layer with its exchange with the adjoining volume phases taken into account. The passage to the limit $h_v \rightarrow \infty$ cannot possibly be realized in this case because the problem (6) degenerates here and turns out to be undefined.

For simplification, surface elasticity and viscosity effects, as well as the surface diffusion effects were not taken into account in (5) and (6). The role of the effects mentioned was investigated in sufficient detail in [3, 13] say. The initial conditions need not be considered in (5) and (6) since periodicity conditions are actually used later.

Let us consider ordered convective flows of small, but finite, amplitude when it is allowable to represent any quantity dependent on the time and coordinates in the form [11]

$$\begin{aligned} \varphi &= \varepsilon^2 \Phi_0 + \sum_{n=1}^{\infty} \varepsilon^n (\Phi_n e^{in\theta} + \Phi_n^* e^{-in\theta}), \\ \theta &= \varepsilon^2 \omega t - kx, \quad \Phi_1 = \Phi_{10} + \varepsilon^2 \Phi_{12} + \dots, \end{aligned} \quad (7)$$

where Φ_n depend only on z while the wave number k is considered real. The quantity ω is also considered real in stationary modes. The parameter ε is introduced for convenience in extracting terms of different order in the amplitude of the fundamental (first) harmonic; at the end of the calculations we should set $\varepsilon = 1$. The quantity ω in (7) is considered small at once, which is confirmed entirely by the subsequent calculations.

Substituting expansions of the type (7) for the stream function and concentration in problems (5) and (6) and separating quantities of different orders in ε in the latter, we arrive at a series of problems of different approximations to determine the coefficients in the mentioned expansions by a standard means. Solving the equations of such problems, obtained from (5), and taking account of the condition at infinity and the disappearance of the normal velocity component on the surface $z = 0$, we arrive at the following representations for the expansion coefficients of the velocity component in the upper layer (to simplify the writing, we omit the subscript 1 corresponding to this layer in the notations) [11]:

$$\begin{aligned} V_{x0} = V_{z0} &= 0, \quad V_{x10} = B(1 - kz)e^{-kz}, \quad V_{z10} = ikBze^{-kz}, \\ V_{x2} &= B'(1 - 2kz)e^{-2kz}, \quad V_{z2} = 2ikB'ze^{-2kz}, \\ V_{x12} &= -\frac{i\omega}{4kv} Bz(2 - kz)e^{-kz}, \quad V_{z12} = \frac{\omega}{4v} Bz^2 e^{-kz}. \end{aligned} \quad (8)$$

Completely analogously, after calculations we obtain for the coefficients of the concentration expansion (7) in the upper liquid layer with conditions on its upper boundary taken into account (we again omit the subscript 1)

$$\begin{aligned} C_{10} &= \left(A + \frac{i\beta}{4kD} Bz \right) e^{-kz} + \dots, \\ C_0 &= C(z-h) + \frac{1}{2D} \left[i(AB^* - A^*B) - \frac{5\beta}{4k^2D} BB^* \right] \left(\frac{1}{2k} + z \right) e^{-2kz} + \dots, \\ C_2 &= \left[A' + \frac{z}{4D} \left(iAB + \frac{i\beta}{2k} B' + \frac{\beta}{32k^2D} B^2 \right) \right] e^{-2kz} + \dots, \\ C_{12} &= \frac{z}{2kD} \left[-i\omega A + \frac{\omega\beta}{4k^2} \left(\frac{1}{D} + \frac{1}{2v} \right) B - \frac{i}{2} BC \right] e^{-kz} + \\ &+ \left\{ \frac{i}{8kD} \left[A'B^* - 2A^*B' + \frac{21i\beta}{16k^2D} B^*B' - \frac{9i}{16kD} ABB^* + \right. \right. \\ &\left. \left. + \frac{1175\beta}{512k^3D^2} B^2B^* + \frac{7i}{8kD} A^*B^2 \right] + \frac{iz}{2D} \left[A'B^* - A^*B' + \right. \right. \end{aligned} \quad (9)$$

$$\left. + \frac{3i\beta}{4k^2D} B^*B' - \frac{3i}{16kD} ABB^* + \frac{565\beta}{512k^3D^2} B^2B^* + \frac{3i}{8kD} A^*B^2 \right\} e^{-3kz} + \dots \quad (9)$$

Terms containing z to powers above the first, which do not affect the equations to determine the constants of integration that follow from the boundary conditions not used in (5) and (6), are denoted by the three dots. Exponentially small quantities proportional to $\exp(-nkh)$ were omitted in (9) in conformity with the above. The definition of c° from (2) and the relationships for the expansion coefficients for the velocity components from (8) were used to obtain (9) from (6).

Corresponding results for the lower liquid layer are obtained formally from (8) and (9) upon changing the sign of the quantities ω , k , h , and i . Therefore, the relations (8) and (9) determine the solution of the problems (5) and (6) in both liquid layers in the form of the expansions (7) in which only the nearest side harmonics are taken into account in addition to the fundamental. This corresponds to keep terms of order not higher than the third in powers of ε in all equations. The relationships (8) and (9) for both layers include ten integration constants A_j , A_j^* , B_j , B_j^* , and C_j ($j = 1, 2$), which should be determined from the remaining unused continuity conditions for the tangential velocity component and the balance of the tangential stress on the interfacial surface in (5) and the coupling conditions between the bulk concentration on both sides of this surface and the balance of the impurity mass in the surface layer in (6). Three equations for the constants follow from each of the conditions mentioned, and correspond to the zeroth, first, and second harmonics being considered. Two of the equations for the zeroth harmonic here degenerate into the identity $0 = 0$. In particular, it follows from the continuity of the tangential velocity component that $B_1 = B_2 = B$ and $B_1^* = B_2^* = B^*$, which we take into account in writing the remaining equations.

The condition imposed on the bulk concentrations on the interfacial surface in (6) results in the following three equations

$$\begin{aligned} m_1 \left\{ -h_1 C_1 + \frac{1}{4kD_1} \left[i(A_1 B^* - A_1^* B) - \frac{5\beta_1}{4k^2 D_1} B B^* \right] \right\} &= m_2 \left\{ h_2 C_2 + \frac{1}{4kD_2} \left[i(A_2 B^* - A_2^* B) + \frac{5\beta_2}{4k^2 D_2} B B^* \right] \right\} = \delta\gamma, \\ m_1 \left\{ A_1 + \frac{\varepsilon^2}{8kD_1} \left[i(A_1' B^* - 2A_1^* B') - \frac{21\beta_1}{16k^2 D_1} B^* B' + \frac{9}{16kD_1} A_1 B B^* + \right. \right. \\ &\quad \left. \left. + \frac{1175i\beta_1}{512k^3 D_1^2} B^2 B^* - \frac{7}{8kD_1} A_1^* B^2 \right] \right\} = m_2 \left\{ A_2 + \frac{\varepsilon^2}{8kD_2} \left[i(A_2' B^* - \right. \right. \\ &\quad \left. \left. - 2A_2^* B') + \frac{21\beta_2}{16k^2 D_2} B^* B' + \frac{9}{16kD_2} A_2 B B^* - \frac{1175i\beta_2}{512k^3 D_2^2} B^2 B^* + \frac{7}{8kD_2} A_2^* B^2 \right] \right\} = \Gamma, \quad m_1 A_1' = m_2 A_2' = \Gamma'. \end{aligned} \quad (10)$$

The quantities $\delta\gamma$, Γ and Γ' with the sense of complex amplitudes of the zeroth, first, and second harmonics of the perturbation γ in the surface concentration are introduced here. The quantity $\delta\gamma$ evidently describes the change in the mean concentration as compares with its unperturbed value γ° defined in (3).

The condition for balance of the tangential stress from (5) yields

$$\left(2\mu^+ + \frac{i\varepsilon^2\omega}{2k^2} \rho^+ \right) B = ia\Gamma, \quad 2\mu^+ B' = ia\Gamma'; \quad (11)$$

in this case the equation for the zeroth equation degenerates.

The condition of impurity mass balance from (6) results in the equations

$$\begin{aligned} D_1 C_1 &= D_2 C_2, \\ ik\gamma^\circ B + \varepsilon^2 \left\{ -i\omega\Gamma + ik(\Gamma^* B' + \Gamma' B^*) + ikm_1 \left[-h_1 C_1 + \frac{i}{4kD_1} \times \right. \right. \\ &\quad \left. \left. \times (A_1 B^* - A_1^* B) - \frac{5\beta_1}{16k^2 D_1^3} B B^* \right] B \right\} = kR\Gamma + \frac{i\beta^-}{4k} B + \\ &\quad + \varepsilon^2 \left\{ -\frac{ir}{4} \Gamma' B^* + \frac{3g}{64k} \Gamma B B^* - \frac{3q}{64k^2} B' B^* - \frac{g}{32k} \Gamma^* B^2 + \right. \end{aligned}$$

$$+ \frac{i\omega r}{2k} \Gamma + \frac{\omega S}{8k^3} B - \frac{i}{4k} (C_2 - C_1) B - \frac{45in}{2048k^3} B^2 B^* \Big], \quad (12)$$

$$i\gamma^\circ B' + i\Gamma B = R\Gamma' - \frac{ir}{8k} \Gamma B + \frac{i\beta^-}{16k^2} B' + \frac{q}{256k^3} B^2.$$

Here we introduced the quantities

$$\begin{aligned} \mu^+ &= \mu_1 + \mu_2, \quad \rho^+ = \rho_1 + \rho_2, \quad \beta^- = \beta_2 - \beta_1, \\ r &= \frac{1}{m_1} + \frac{1}{m_2}, \quad g = \frac{1}{m_1 D_1} + \frac{1}{m_2 D_2}, \quad q = \frac{\beta_2}{D_2} - \frac{\beta_1}{D_1}, \\ n &= \frac{\beta_2}{D_2^2} - \frac{\beta_1}{D_1^2}, \quad R = \frac{D_1}{m_1} + \frac{D_2}{m_2}, \\ S &= \beta_2 \left(\frac{1}{D_2} + \frac{1}{2\nu_2} \right) - \beta_1 \left(\frac{1}{D_1} + \frac{1}{2\nu_1} \right). \end{aligned} \quad (13)$$

Let us consider the general principles of the analysis of the system (10)-(12) for the constants of integration [11]. It is already evident from (7) that the amplitudes of the fundamental harmonics are the sum of terms of the order of ε and ε^3 within the limits of the accuracy taken. The amplitudes of the zeroth and second harmonics are on the order of ε^2 . Consequently, only terms of order ε , but not ε^3 , whose definition corresponds to linear stability theory, need be taken into account in their representation from the appropriate equations in (10)-(12) in terms of the first harmonic amplitudes in the latter. We obtain from the equations in (10) and (11) corresponding to the first harmonic, to the accuracy mentioned.

$$A_j = -\frac{2i\mu^+}{am_j} B, \quad \Gamma = -\frac{2i\mu^+}{a} B, \quad j = 1, 2 \quad (14)$$

(hence and from the second equation in (12) written with terms of order ε^2 , neglected the equation for the neutral stability curve can evidently be obtained).

Substituting (14) into the first equation in (10) and (12), we obtain an expression for the nonzero amplitudes of the zeroth harmonic:

$$\begin{aligned} C_1 &= -\frac{D_2}{m_1 h_1 D_2 + m_2 h_2 D_1} \left[\frac{\mu^+}{ak} \left(\frac{1}{D_2} - \frac{1}{D_1} \right) + \frac{5}{16k^3} \left(\frac{\beta_2 m_2}{D_2^2} + \frac{\beta_1 m_1}{D_1^2} \right) \right] \delta, \quad C_2 = \frac{D_1}{D_2} C_1, \quad \delta = BB^* = |B|^2, \\ \delta\gamma &= \frac{1}{m_1 h_1 D_2 + m_2 h_2 D_1} \left[\frac{\mu^+}{ak} (m_1 h_1 + m_2 h_2) + \frac{5m_1 m_2}{16k^3} \left(\frac{\beta_2 h_1}{D_2} + \frac{\beta_1 h_2}{D_1} \right) \right] \delta. \end{aligned} \quad (15)$$

Representations for the amplitudes of the second harmonic follow analogously from the last equations in (10)-(12)

$$A_j' = \frac{\Gamma'}{m_j}, \quad B' = \frac{ia}{2\mu^+} \Gamma', \quad \Gamma' = \left[\frac{2\mu^+}{a} \left(1 + \frac{r}{8k} \right) - \frac{q}{256k^3} \right] \left[R + \frac{a}{2\mu^+} \left(\gamma^\circ - \frac{\beta^-}{16k^2} \right) \right]^{-1} B^2, \quad j = 1, 2. \quad (16)$$

Now, keeping in mind the analysis of the slightly nonlinear problem under consideration, we use (15) and (16) in the equations for the first harmonic in (10)-(12) taking all the terms written down in them into account. As a result of calculations we obtain

$$\begin{aligned} A_j &= \left\{ -\frac{2i\mu^+}{am_j} + \frac{\omega\rho^+}{2am_j k^2} - \frac{i\delta}{8kD_j} \left[\frac{3}{m_j} \left(1 - \frac{7am_j\beta_j}{32\mu^+ D_j k^2} \right) \times \right. \right. \\ &\times \left. \left(\frac{2\mu^+}{a} \left(1 + \frac{r}{8k} \right) - \frac{q}{256k^3} \right) \left(R + \frac{a}{2\mu^+} \left(\gamma^\circ - \frac{\beta^-}{16k^2} \right) \right)^{-1} + (-1)^j \frac{23\mu^+}{8am_j D_j k} \left(1 - \frac{1175am_j\beta_j}{1472\mu^+ D_j k^2} \right) \right\} B, \quad (17) \\ \Gamma &= \left(-\frac{2i\mu^+}{a} + \frac{\omega\rho^+}{2ak^2} \right) B, \quad j = 1, 2, \end{aligned}$$

as well as the solvability condition for the system of equations of the first harmonic considered as linear with coefficients dependent not only on the physical regime parameters and k but also on δ (i.e., the condition for the presence of a nontrivial solution $B \neq 0$ for $\delta = \text{const}$):

$$i\omega f_1(k) = f_2(k) + \delta f_3(k). \quad (18)$$

Here we introduced the following functions of k :

$$\begin{aligned} f_1(k) &= 2 \left(1 + \frac{r}{2k} \right) \mu^+ + \frac{R\rho^+}{2k} + \frac{Sa}{8k^3}, \\ f_2(k) &= -2R\mu^+k - a \left(\gamma^\circ - \frac{\beta^-}{4k^2} \right) k, \\ f_3(k) &= -\frac{a}{4} \left(r + \frac{3qa^2}{32\mu^+k^2} \right) \left[\frac{2\mu^+}{a} \left(1 + \frac{r}{8k} \right) - \frac{q}{256k^3} \right] \left[R + \frac{a}{2\mu^+} \left(\gamma^\circ - \frac{\beta^-}{16k^2} \right) \right]^{-1} - \frac{5g\mu^+}{32k} - \frac{45na}{2048k^3} \\ &\quad - \frac{1}{m_1h_1D_2 + m_2h_2D_1} \left\{ \mu^+ (m_1h_1 + m_2h_2) + \frac{5m_1m_2a}{16k^2} \left(\frac{\beta_2h_1}{D_2} + \frac{\beta_1h_2}{D_1} \right) + \right. \\ &\quad \left. + (D_2 - D_1) \left[\frac{\mu^+}{4k^2} \left(\frac{1}{D_2} - \frac{1}{D_1} \right) + \frac{5a}{64k^4} \left(\frac{m_2\beta_2}{D_2^2} + \frac{m_1\beta_1}{D_1^2} \right) \right] \right\}. \end{aligned} \quad (19)$$

We took $\varepsilon = 1$ at once in (17) and (19). Since it was actually assumed that the Reynolds number is much less than the Peclet number (which was the basis for neglecting nonlinear terms in the Navier-Stokes equations while conserving such terms in the convective diffusion equations), then we should take $v_j \gg D_j$ and should set $S = q$ in (13).

If $i\omega = \alpha + i\Omega$, then it is easily seen from (18) and (19) that the frequency of Ω of the convective motion is identically zero, and the increment is

$$\alpha = f_1^{-1}(k) [f_2(k) + \delta f_3(k)]. \quad (20)$$

Within the framework of linear theory, we should set $\delta = 0$ in (20), i.e., the neutral stability curve is determined by the equality $f_2/f_1 = 0$.

Let $a > 0$. We consider the case $\beta^- = \beta_2 - \beta_1 > 0$ when the impurity flux is directed from the layer with the smaller to the layer with the higher diffusion coefficient (for definiteness, we consider the β_j positive). Then taking account of (1) and (13) it is easy to see that $S = q > 0$, i.e., it follows from (19) that $f_1 > 0$. In this case the state of rest is unstable relative to perturbations whose wave number is less than the critical value:

$$k < k_*, \quad k_* = \frac{1}{2} \left(\frac{\beta_2 - \beta_1}{\gamma^\circ + 2Ra^{-1}\mu^+} \right)^{1/2}, \quad (21)$$

which agrees with the classical result (see [2, 13], say).

If $\beta^- = \beta_2 - \beta_1 < 0$ holds for $a > 0$, then $D_2 > D_1$ and $S = q < 0$. In this case $f_2 < 0$ for all $k > 0$ and f_1 changes sign for $k = k^*$ while remaining positive in the domain $k > k^*$. The equation

$$k^{*3} + (r\mu^+ + R\rho^+/2) k^{*2} - a|q|/8 = 0 \quad (22)$$

for k^* follows from (19), and as is easy to show, has a unique positive real root. The instability in this case holds for $k < k^* \sim k^*$; the critical value k^* does not determine the boundary of the stability domain exactly because we have $f_1 \approx 0$ for $k \approx k^*$, i.e., α , and therefore also ω cannot possibly be considered small in opposition to the assumption in (7). For an analogous reason the proposed theory does not generally yield conditions for the onset of instability relative to perturbations with a nonzero (or in any case not small) frequency Ω .

For complete definiteness of the results obtained it is necessary to find k and δ , which had been considered as parameters up to now. We obtain the equations for them from the

conditions of stationary of the ordered convective structure under consideration as was proposed in [11, 14], i.e., we require the increment α from (20) and its derivative with respect to k to be zero for $\delta = \text{constant}$. We have

$$f_2(k) + \delta f_3(k) = 0, \quad f_2'(k) + \delta f_3'(k) = 0, \quad (23)$$

where the prime denotes differentiation with respect to k . The extremum α should be the maximum [11, 14].

Investigation of (23) in application to specific systems of different kinds is fraught with very tedious calculations and is an independent problem. Here we just mention that the theory proposed should be valid in the case the values of k and δ obtained from (23) satisfy the inequalities $k > 0$, $0 < \delta \ll 1$. If it turns out that $\delta \geq 1$ then this theory is at least not exact, if $\delta < 0$, then the assumption taken about the establishment of an ordered convective structure is apparently not valid and actually results in the generation of chaotic motion, i.e., interphasal turbulence, for appropriate value of the Marangoni instability parameters.

Let us mention the presence of an original scale effect in the system. As is seen from (19), the form of the function f_3 depends substantially on the relationship of the layer thicknesses even for $h_j \gg k^{-1}$, where k^{-1} is the characteristic dimension of the structure. This means that the linear scale and intensity of the convective motion depend substantially on firstly the thickness of the liquid layers even if their main volume is not entrained in the motion, and secondly, on the features of the diffusion flux organization in the unperturbed system (the kind of boundary conditions on the layer outer boundaries).

In conclusion, we mention that all the equations obtained can be reformulated in application to an analogous problem on thermocapillary convection. In this case, we have a continuous temperature field instead of the unperturbed concentration fields (2) and we introduce the temperature dependence of the surface tension coefficient in place of (4). The relationships (3) are formally valid for $m_j = 1$ but the introduction of surface temperature as an additional variable is inexpedient. The heat-conduction (in (1) and in the last condition in (6)) or thermal diffusivity (in the equations from (6)) coefficients must be considered in place of D_j . In this case the problem (5) retains its form upon replacement of γ by T_s in the boundary conditions, and the problem (6) changes in the respect that temperature and heat flux continuity conditions are given simply on the $z = 0$ plane. The relationships for the amplitudes of the different quantities are completely analogous to (15)-(17), Eqs. (18), (20), and (23) retain their form but with other functions $f_j(k)$ that differ from (19). Because of insufficient space, the relationships for the thermocapillary slightly nonlinear convection are not written down here.

NOTATION

α , a coefficient in (4); A, A', B, B', C , integration constants; c , volume concentration; D , diffusion coefficient; f , functions defined in (19); h , liquid layer thickness; k , wave number; m , constant of sorption equilibrium; g, n, q, R, r, S , parameters defined in (13); t , time; v , velocity; x, z , coordinates; α , increment in vibration growth; β , concentration gradient; γ , surface concentration; δ , amplitude squared; ϵ , a fictitious smallness parameter; μ, ν , dynamic and kinematic viscosity; ρ , density; σ , surface tension coefficient; ψ , stream function; Ω, ω , real and complex frequencies. Subscripts; layer number or harmonic number; $s, z = 0$ interface; degree is the unperturbed state of rest; amplitudes of the variables are denoted by appropriate capital letters.

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STABILITY CRITERION OF THREE-DIMENSIONAL PERTURBATIONS ON CONCAVE
ELASTIC SURFACES

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Stability criteria are determined experimentally for the boundary layer on concave elastic surfaces in the preturbulent transition region.

As always, the investigation of the physical processes of turbulent boundary-layer formation remains urgent for a broad class of scientific and practical problems. The structure of the perturbing motion in different stages of laminary boundary-layer transition into turbulent and the process of transformation of plane into three-dimensional perturbations are studied experimentally in [1], in longitudinal vortices of the Benny-Lin type on a flat plate, and in Goertler vortices on a curvilinear plate. The general features and regularities of formation and existence of these vortical systems are determined and the Goertler neutral curve is first constructed by experimental means.

The possibility of controlling the hydrodynamic stability by using different elastic plates is studied in [2], and three-dimensional perturbations on a rigid plate in [3].

The purpose of the present paper is to investigate the influence of an elastic surface on the stability of longitudinal vortices and to determine the possibility of controlling three-dimensional perturbations, particularly, for heat- and mass-transfer problems.

The methodology of the experiment is based on the susceptibility of the boundary layer to different perturbations [1, 4, 5]. According to these representations, under ideal flow conditions prerequisites exist for the generation of perturbations in the form of Tollmien-Schlichting waves with their subsequent transformation into more complex types. Factors degrading the hydrodynamic stability (for instance, the high degree of main flow turbulence, streamlined surface roughness, etc.) result in magnification of the perturbing motions. The greater the intensity and quantity of the degrading factors, the more rapidly do natural boundary layer perturbations develop. The susceptibility problem consists of studying the nature of the interaction at different stages of boundary layer transition between existing natural perturbations and those induced from outside.

On the basis of this definition the linear and Goertler instabilities are particular cases of the susceptibility problem whose investigation must be conducted under the greatest possible ideal fluid flow conditions and the induction of small perturbations. On the one hand, by this the influence of uncontrollable factors and nonlinear interaction of the induced and natural perturbations on stability is eliminated, and on the other, the possibility is achieved of determining the boundary-layer reaction to perturbations of a given scale. The induced small plane perturbations magnify and interact with the natural perturbations in the linear stability investigation. Small three-dimensional perturbations, which excite the existing plane natural perturbations and result in their rapid transformation into three-dimensional perturbations that interact with the induced perturbations are introduced into the boundary layer in the study of the Goertler instability.

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